

Proposed changes to *Oligomorphic Permutation Groups*

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These proposed changes were marked in the text or written on two loose sheets in the author's copy of the book. They are typed up as found but with some annotations in square brackets and references added at the end.

Addenda

1. Add *Research Problem* on p.18: Find estimates for the function involved in the finite form of (1.10).
2. *Dixon's Theorem* [2]: almost all 2-generator subgroups of the symmetric group on a countable set are free and highly transitive. [This is an infinite analogue of his famous theorem that the probability that two elements of a finite symmetric group S_n generate S_n or A_n tends to 1 as $n \rightarrow \infty$.]
3. Exercise 4.2.1 [This asks "Find two order-automorphisms of \mathbb{Q} which generate a free group"]: make more explicit:
 - (i) Show that, for any non-empty reduced word w in x and y and their inverses, there exist order-preserving permutations g, h of \mathbb{Q} such that $w(g, h) \neq 1$.
 - (ii) Using this, and the fact that \mathbb{Q} and $\mathbb{Q} \cap (\alpha, \alpha + 1)$ are order-isomorphic (where α is irrational), show that the free group of rank 2 is embedded in $\text{Aut}(\mathbb{Q}, <)$.
4. §3.3, Example 4: give more details here! [More on the asymptotics of $(f_n(G))$ can be found in a paper of Cameron, Prellberg and Stark [1].]
5. §3.4, Exercise 1: Hence show that, for this group, f_n is roughly $q^{n^2/2}$.
6. p61, *Problem 1*: Show that other expressions such as $f_n(G)/n^d$ ($d \in \mathbb{Z}$) or $\log \log f_n(G)/\log n$ have limits as $n \rightarrow \infty$. Prove general results about smoothness.

Corrections

I have not included corrections of typos here.

p13: A subset of X is dense if it meets every *non-empty* open set.

p27: No reference to David Evans

p29: $|G : H|$ should be $|H : G|$ in the proof of 2.8

p29: Commas missing in Exercise 1

p38, Ex. 3, Step 1: $\alpha \sim \beta$ should be $\alpha \sim \alpha g$.

References

- [1] P. J. Cameron, T. Prellberg and D. Stark, Asymptotic enumeration of 2-covers and line graphs, *Discrete Math.* **310** (2010), 230–240; doi: 10.1016/j.disc.2008.09.008
- [2] J. D. Dixon, Most finitely generated permutation groups are free, *Bull. London Math. Soc.* **22** (1990), 222–226.